Master's Degree Numerical Methods in Engineering

Computational Structural Mechanics and Dynamics

# Assignment 6 , 7: Beams and Plates

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# 1 Beams

a) Program In Mat Lab the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix

$$K_s^e = \frac{(GA)^e}{l}\begin{bmatrix} 1 & l/2 & -1 & l/2 \\ l/2 & l^2/4 & -l/2 & l^2/4 \\ -1 & -l/2 & 1 & -l/2 \\ l/2 & l^2/4 & -l/2 & l^2/4 \end{bmatrix}$$

b) Solve the following problem with a 64 element mesh with the

- 2 nodes Euler Bernulli element
- 2 nodes Timoshenko Full Integrate element
- 2 nodes Timoshenko Reduce Integration element.

Compare maximum displacements, moments and shear for the 3 elements against the a/L relationship.

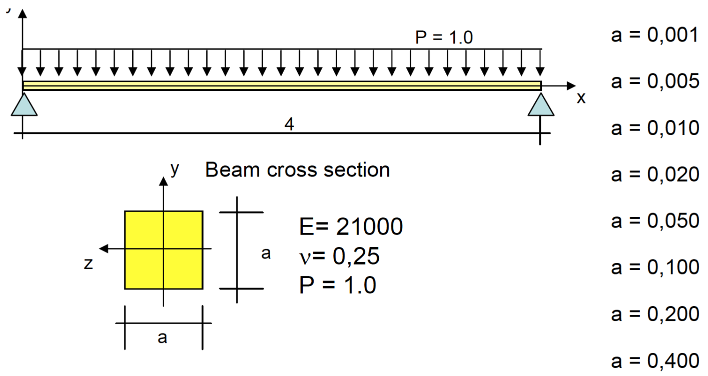


Figure 1: Problem geometry and required areas.

a) For coding the reduced integration Timoshenko we should change the shear stiffness in the code to the given K matrix. So the $k_s$ in the code will be changed to:

$$K_s^e = \frac{(GA)^e}{l} \begin{bmatrix} 1 & l/2 & -1 & l/2 \\ l/2 & l^2/4 & -l/2 & l^2/4 \\ -1 & -l/2 & 1 & -l/2 \\ l/2 & l^2/4 & -l/2 & l^2/4 \end{bmatrix}$$

b) We start the problem by first defining the problem in the GID. We define the 4m beam using a line and the boundary conditions of fixed x-displacement in the both ends of the beam with a 1 N/m uniform load on the y-direction normal to the beam. We get the input file for the matfem and go to the code. We use the input code with the main code and extract the solutions from the .res output file. For each a the input file should be modifies manually; changing the area and inertia of the problem. The needed inertia and area are given in the table below:

| a | area | inertia | a/l |
|---|---|---|---|
| 0.001 | 1.00E-06 | 8.33E-14 | 0.00025 |
| 0.005 | 2.50E-05 | 5.21E-11 | 0.00125 |
| 0.01 | 1.00E-04 | 8.33E-10 | 0.0025 |
| 0.02 | 4.00E-04 | 1.33E-08 | 0.005 |
| 0.05 | 2.50E-03 | 5.21E-07 | 0.0125 |
| 0.1 | 1E-02 | 8.33E-06 | 0.025 |
| 0.2 | 4E-02 | 1.33E-04 | 0.05 |
| 0.4 | 1.6E-01 | 2.13E-03 | 0.1 |

Figure 2: Table of needed areas and inertias

After evaluating each step for the three methods of Euller, Timoshenko and recudes integration Timoshenko the values for the maximum x-displacement, moment and shear were written and the figures below represent the differences in the three methods.

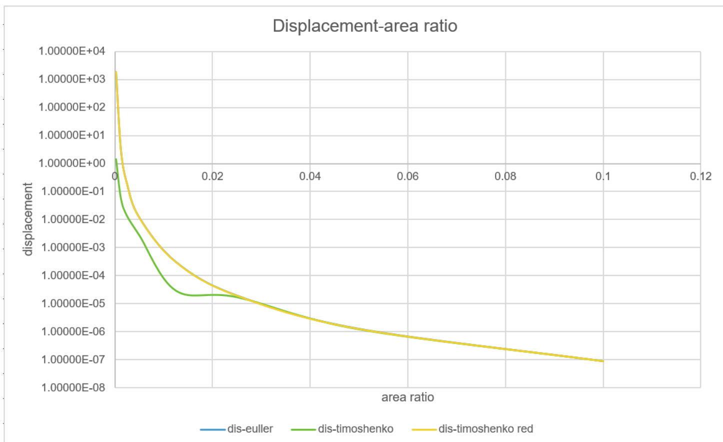


Figure 3: Maximum Displacement

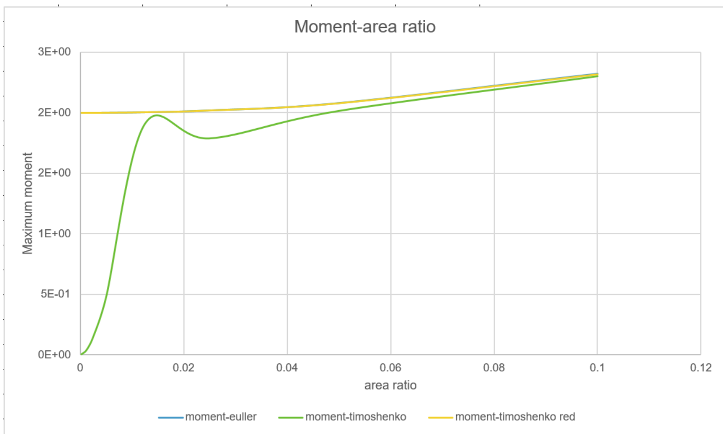


Figure 4: Maximum Moments

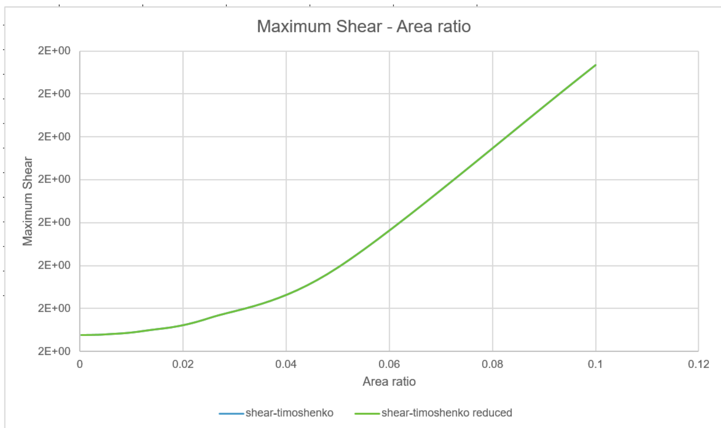


Figure 5: Maximum Shear

As guessed before we can see that both for the displacement and the moment the answers of the Timoshenko method for smaller 'a's is very far from the real solution. We know that the Euller method has no error in the displacement so the Timoshenko has considerable error in the displacement. A similar case can be seen for the maximum moments obtained. In case of the reduced integration Timoshenko it is seen that the values obtained have small differences with that of the Euller but the difference is very small and as seen form the figures the difference is not considerable.

# 2 plates

a) Analyze the shear blocking effect on the Reissner Mindlin element and compare with the MZC element. For the Simple Support Uniform Load square plate. Use the 5x5 Mesh.

b) Define and verify a patch test mesh for the MCZ element

a) For this case first the matfem programs were downloaded from the website. GID was used in order to create the create the input file. The domain is a one by one square with 5 elements in each direction. A vertical uniform load is applied to the plate and the edges have a fixed displacement in the z-direction. The material used has the data: E = 1.092e+07, $\nu = 0.3$. Five different cases of thicknesses varying from 0.001 to 0.4 meters have been assumed for the model. In each step the value of the thickness has changed and the maximum displacement in the z-direction and the maximum moments are recorded in each step. The displacement and the moments for the MCZ and the R-M method are given below:

| thickness | displacement | moment | | |
|---|---|---|---|---|
| 0.001 | -2.38801E+01 | -1.47201e-01 | -4.89054e-03 | -2.03324e-02 |
| 0.01 | -2.40949E-02 | -1.48524e-01 | -4.93451e-03 | 2.05152e-02 |
| 0.02 | -3.04168E-03 | -1.49995e-01 | -4.98337e-03 | 2.07183e-02 |
| 0.1 | -2.62419E-05 | -1.61759e-01 | -5.37422e-03 | 2.23433e-02 |
| 0.4 | -5.21856E-07 | -2.05875e-01 | -6.83992e-03 | 2.84369e-02 |

Figure 6: The data of the MCZ method

| thickness | displacement | moment | | |
|---|---|---|---|---|
| 0.001 | -2.23743E+01 | -1.46110e-01 | -3.89179e-02 | 1.87443e-02 |
| 0.01 | -2.26031E-02 | -1.47215e-01 | -3.90792e-02 | 1.88865e-02 |
| 0.02 | -2.86382E-03 | -1.48052e-01 | -3.89076e-02 | 1.89953e-02 |
| 0.1 | -2.71611E-05 | -1.47970e-01 | -3.14265e-02 | 1.90118e-02 |
| 0.4 | -1.15170E-06 | -1.71773e-01 | -2.50887e-02 | 2.21113e-02 |

Figure 7: The data from the R-M method

From the obtained values for the displacement and the created moments in the plate we can see that for the small thicknesses the value of the displacements recorded are much higher than the values that a plate with this material would be able to withstand so for the small thicknesses it is obvious that the plates will break before reaching the values. In the values for the moment we can see that there is a significant difference which is because of the shear energy considered in the R-C method. In the MCZ method we can see that after the thickness increases significantly the difference between with the RM method increases, the reason is that because of the growing thickness the effect of the shear moment becomes more and more considerable and because the MCZ method does not consider its effects we see a growing difference between the two.

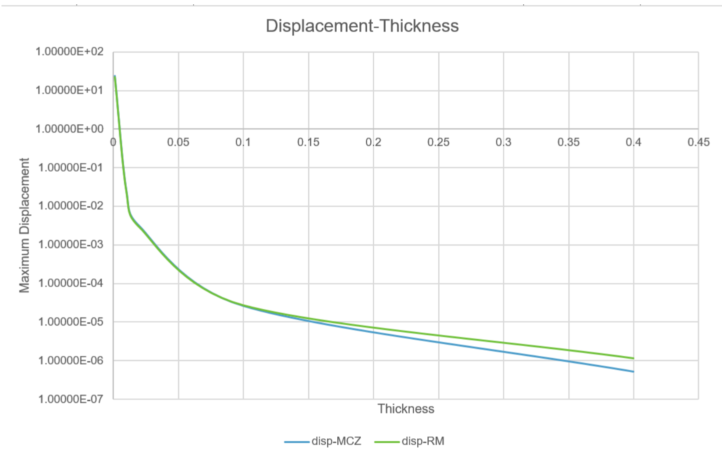


Figure 8: Maximum Displacement

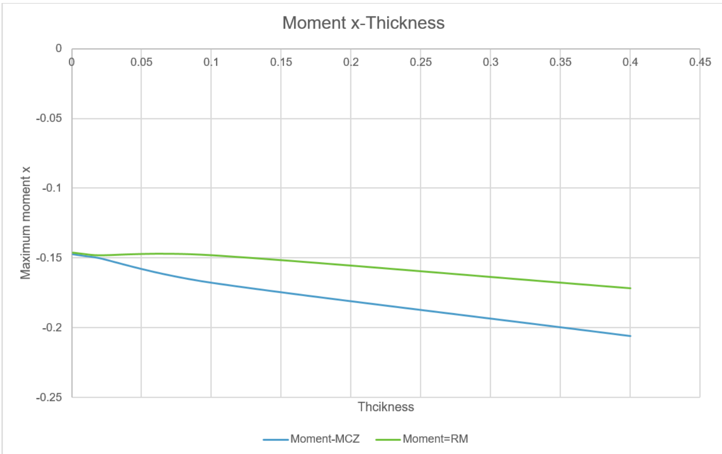


Figure 9: Maximum Moment x

b) The patch test is a method used to verify the correctness and convergence of the element used in the finite element method. This method is based on assuming a field of displacement for the nodes in the mesh and applying the values of the field only to the nodes on the boundary and starting the simulation, in case the values obtained for the nodes in the center is the same as the values reached by the assumed displacement field then the mesh satisfies the patch test. For our case a simple domain of a square of one by one is considered with four similar linear quadrilateral elements. Two case of displacement field are used for this test. The first one a simple case of a displacement field of (x+y)10e-02 and another case used by a paper in the MIT for a patch test of quadrilateral *elements*[1]. The displacement field for this case is given by $(x^2 + xy + y^2)(10e - 03)/2$. In this case only the displacement field in the z=direction was used and the conditions for the slopes were not considered.

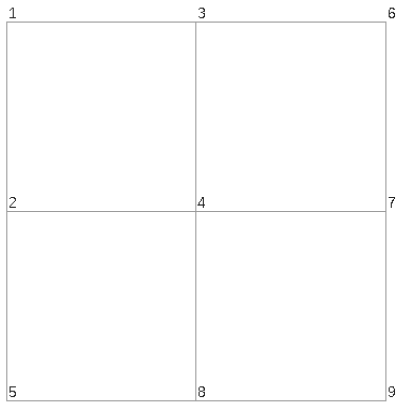


Figure 10: The domain used for the patch test

As seen in the figure the only value to be checked is the value of the displacement on the fourth node. For the first case of the displacement field the displacement at the fourtch node should be 1.00e-02 so after running the model we obtain:

```
Result "Displacement" "Load Analysis"  1  Vector OnNodes
ComponentNames "X-Displ", "Y-Displ", "Z-Displ"
Values
     1 0.0 0.0  -1.00000e-02
     2 0.0 0.0  -1.50000e-02
     3 0.0 0.0  -5.00000e-03
     4 0.0 0.0  -1.00005e-02
     5 0.0 0.0        00000
     6 0.0 0.0  -2.00000e-02
     7 0.0 0.0  -5.00000e-03
     8 0.0 0.0  -1.50000e-02
     9 0.0 0.0  -1.00000e-02
End Values
#
```

Figure 11: Displacements obtained with the first field

We can see that the value at node 4 is exactly 1.00e-02 , with a very small error, so the model satisfies the patch test. As the first case if we do the same calculations for the second displacement field the value at node 4 should be 3.75e-04 and using the MATFEM we obtain:

```
Result "Displacement" "Load Analysis"  1  Vector OnNodes
ComponentNames "X-Displ", "Y-Displ", "Z-Displ"
Values
     1 0.0 0.0   5.00000e-04
     2 0.0 0.0   8.75000e-04
     3 0.0 0.0   1.25000e-04
     4 0.0 0.0   4.83354e-04
     5 0.0 0.0         00000
     6 0.0 0.0   1.50000e-03
     7 0.0 0.0   1.25000e-04
     8 0.0 0.0   8.75000e-04
     9 0.0 0.0   5.00000e-04
End Values
#
```

Figure 12: Displacements obtained with the second field

In this case the value obtained is 4.83e-04, in this case the error increases significantly.

# 3 References


- https://abaqus-docs.mit.edu/2017/English/SIMACAEVERRefMap/simaver-c-platebendpatch.htmsimaver-c-platebendpatch-t-inputfiles1[1]
- Beams PDF of lecture
- Plates PDF of lecture